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Influence of the tritium β^- decay on low-temperature thermonuclear burn-up in deuterium-tritium mixtures

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Low-temperature ($T \leqslant 15$ keV) thermonuclear burn-up in deuterium-tritium mixtures with various deuterium–tritium–helium-3 ratios is considered. The general dependence is studied for the critical burn-up parameter $x_c = \rho r_c$ upon the initial temperature T, density ρ_0, and tritium molar concentration y for the $[\mathrm{D}]{:}y[\mathrm{T}]{:}(1-y)[{}^3\mathrm{He}]$ mixture. In particular, it is shown that, if the tritium concentration y decreases, then the critical burn-up parameter $x_c(T, \rho_0, y)$ grows very quickly (at fixed T and ρ_0). This means that tritium β^- decay significantly complicates thermonuclear burn-up in deuterium-tritium mixtures.

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In this paper, we consider low-temperature ($T \leqslant 15$ keV) thermonuclear burn-up in deuterium-tritium mixtures with various deuterium–tritium–helium-3 ratios. This problem is of practical interest, since such a situation can be found in equimolar DT mixtures, when the tritium β^- decay [1] cannot be ignored. The problem is formulated as follows. Initially, one has an equimolar (or 1:1) DT mixture. The tritium β^- decay increases concentration of the ^{3}He nuclei, and therefore the mean atomic charge $\overline{Z}$ [2] in the mixture. In general, thermonuclear burn-up occurs in $[\mathrm{D}]{:}y[\mathrm{T}]{:}(1-y)[{}^3\mathrm{He}]$ mixtures, where $0 \leqslant y \leqslant 1$, rather than in equimolar DT mixtures. Since $\overline{Z} \geqslant 1$ in such mixtures, it is clear that the high-temperature bremsstrahlung I_B loss is larger [and even significantly larger $I_B \sim (\overline{Z})^3$] than in the equimolar DT mixture, i.e., thermonuclear burn-up becomes more complicated (as y decreases to 0). Another reason for the difference in burn-up is the direct decline of the tritium concentration. As is well known (see, e.g., [2,3]), in an equimolar DT mixture at solid hydrogen density ($\rho_0 = 0.213\ \mathrm{g\,cm^{-3}}$), the energy gain from (d,t) fusion reactions exceed high-temperature bremsstrahlung loss at $T_i = T_e \approx 4$ keV. This value is called the ideal ignition temperature T_{id} [4] (at $\rho_0 = 0.213\ \mathrm{g\,cm^{-3}}$). In general, for the $[\mathrm{D}]{:}y[\mathrm{T}]{:}(1-y)[{}^3\mathrm{He}]$ mixtures with $\rho_0 = 0.213\ \mathrm{g\,cm^{-3}}$ the ideal ignition temperature increases very rapidly (up to 30–40 keV and higher) as y decreases. However, by increasing the compression (or density) of the thermonuclear fuel, one can decrease the ideal ignition temperature back to realistic 4–10 keV values. In fact, high compressions reduce the effective bremsstrahlung loss from the hot, combustion zone. Eventually, thermonuclear burn-up in deuterium-tritium mixtures with helium-3 nuclei can proceed at the same (or comparable) temperatures as for equimolar (or 1:1) DT mixtures. However, required densities (or compressions) in this case are significantly larger than for equimolar DT mixtures. Below, we determine the densities needed to compensate and exceed high-temperature bremsstrahlung loss in DT mixtures with various concentrations of ^{3}He nuclei.

As is well known (see, e.g., [4]), thermonuclear burn-up starts when energy release from fusion reactions (in some fuel) exceeds all possible energy losses, and, in particular, the high-temperature bremsstrahlung loss ($\sim \sqrt{T}$). Thermonuclear burn-up is represented by the so-called burn-up equation, which is simply a balance equation between energy release and energy losses. In the general case, such an equation has a very complex form, but here we shall make use of a few simplifying assumptions. First, thermonuclear burn-up is considered in its classical form for a homogeneous, immovable combustible mixture (or thermonuclear fuel). The initial density ρ_0 of the combustible mixture is a constant, i.e., it does not depend on the spatial coordinates. The classical form of the burn-up problem means that at $t=0$ some central part of the mixture is instantaneously heated to very high temperatures. At such temperatures thermonuclear ignition proceeds in the hot central areas, and a thermonuclear burn wave then propagates out, igniting the rest of the fuel. In other words, this is thermonuclear burn-up propagating from a hot, central spot. Second, we restrict our present analysis to the case of spherical geometry only. Spherical geometry means that (1) the initially heated zone has a spherical form $[r(t=0) = r_0]$, and (2) the expanding hot zone will keep such a spherical form at all later moments, i.e., the propagating thermonuclear burn wave has a spherical form. Third, we shall consider only low-temperature burn-up with $T \leqslant 15$ keV. For such temperatures the electron-ion equipartition time τ_{ie} (see, e.g., [5]) is very short, and therefore, the one-temperature approximation can be used, i.e., $T_e = T_i = T$. Furthermore, the nuclear $(d, {}^3\mathrm{He})$ reaction can be ignored at such temperatures and the relatively low densities ($\rho_0 \leqslant 1000\ \mathrm{g\,cm^{-3}}$) that are considered below. Also, at $T \leqslant 15$ keV the nuclear (d,t) reaction proceeds $\approx$59–75 times faster than the competing (d,d) reaction (this will be used below).

Let $r_f(t) \geqslant r_0$ be the radius of the hot, spherical spot created by the shock (thermal) wave at the time t. The velocity of the hot zone expansion is dr_f/dt. In reality, the hot zone expands either by a high-temperature thermal wave $V_t = (dr_f/dt)_t$, or by a detonation wave $V_d = (dr_f/dt)_d$. The actual (or observed) velocity of the hot zone expansion $V_{\max}$ is the larger of the two corresponding velocities, i.e., $V_{\max} = \max\{(dr_f/dt)_d, (dr_f/dt)_t\}$. The temperature behind the shock (or thermal) wave $T = T_f$ is significantly higher than the initial temperature before such a wave, where $T_0 \approx 0$. Below, without loss of generality we shall assume that (1) the temperature T_f behind the shock (or thermal) wave is very high and does not depend on the spatial coordinates, and (2) the initial temperature before the shock (or thermal) wave is zero, i.e., $T_0=0$. Now, the burn-up equation takes a relatively simple, one-dimensional form [3]:

$$\begin{aligned}\mathcal{C}\frac{dT}{dt}&=-\mathcal{C}\frac{3}{r_f}\frac{dr_f}{dt}T+\tilde{q}(r_f,T,\rho_0)\\&=-\mathcal{C}\frac{3}{r_f}V_{\max}T+\tilde{q}(r_f,T,\rho_0),\end{aligned}\tag{1}$$

where T is the temperature in the hot zone and $\mathcal{C}$ is the specific heat (in MJ g^{-1} keV^{-1}, where 1 MJ$=1\times10^6$ J) per unit mass of thermonuclear fuel. In the first approximation the specific heat $\mathcal{C}$ does not depend upon T. In fact, such an approximation has very good accuracy if (1) $T\geqslant0.1$ keV, and (2) the thermonuclear fuel contains only light elements (with $A\leqslant20$ and $Z\leqslant10$). In the last equation $\tilde{q}(r_f,T,\rho_0)$ is the so-called energy release function (per unit volume). By introducing the burn-up parameter $x=\rho_0 r_f$ (see, e.g., [3]), one can rewrite the burn-up equation in the the form (see [3,6])

$$\frac{dT}{dx}=-\frac{3}{x}T+\frac{q(x,T,\rho_0)}{\mathcal{C}V_{\max}},\tag{2}$$

where $q(x,T,\rho_0)=\tilde{q}(r_f,T,\rho_0)/\rho_0$ is the so-called normalized energy release function (per unit mass). In contrast with r_f, the burn-up parameter x does not depend significantly on ρ_0. The explicit expressions for the $q(x,T,\rho_0)$ function and for the $\mathcal{C}$ and $V_{\max}$ values depend significantly on the ionic content of the thermonuclear fuel considered.

Let y be a parameter that designates the relative molar concentration of the remaining tritium nuclei in the [D]:y[T]:$(1-y)$[^{3}He] mixture. Now, the formula for $\mathcal{C}$ takes the form (see [2])

$$\mathcal{C}=\frac{144.716\,44}{\bar{A}}(1+\bar{Z})=144.716\,44\left(1-\frac{y}{5}\right),\tag{3}$$

where $\bar{A}$ and $\bar{Z}$ are the mean atomic mass and mean atomic number (or charge), respectively. They must be expressed in units in which the proton mass and charge equal 1 [7]. Therefore, for the considered [D]:y[T]:$(1-y)$[^{3}He] mixture $\bar{A}=2.5$ always, while $\bar{Z}$ is a linear function of $y(\bar{Z}=1.5-0.5y)$.

As mentioned above the hot zone expands by thermal or detonation waves. The initial, very fast (or high-temperature) stage of such an expansion is always represented as a thermal wave motion. The appropriate analytical formula for the velocity of a thermal wave V_T (in cm nsec^{-1}) takes the form [5]

$$V_T=b_n\frac{\kappa(T)}{\mathcal{C}x}=\frac{b_n}{144.716\,44}\frac{\kappa(T)}{(1-y/5)x},\tag{4}$$

where $\kappa(T)\sim T^n$ is the electron thermal conductivity and x is the burn-up parameter. Since we assume here that $T=T_i=T_e$ and $T\leqslant15$ keV, the ionic part of the total thermal conductivity [2] is negligibly small in comparison with the electron thermal conductivity. For the spherical case considered the parameter b_n takes the form [6]

$$b_n=\frac{2}{\pi^{n/2}n}\left(n+\frac{2}{3}\right)^n\left(1+\frac{2}{n}\right)^n\frac{\Gamma^n(\frac{1}{2}+1/n)}{\Gamma^n(1/n)}.\tag{5}$$

Since in the case considered $n=2.5$ (see below), one finds $b_n=2.390\,15$.

The electron thermal conductivity for a real, classical plasma can be written in the form [2] $\kappa(T)=\kappa_L(T)\varepsilon\,\delta_T$, where $\kappa_L(T)$ is the electron thermal conductivity for a high-temperature, Lorentz plasma [5]:

$$\kappa_L(T)=20\left(\frac{2}{\pi}\right)^{3/2}\frac{(kT)^{5/2}k}{m_e^{1/2}e^4Z\ln\Lambda_{ei}}=0.104\,104\frac{T^{5/2}}{Z\ln\Lambda_{ei}},\tag{6}$$

where T is in keV and $\kappa_L(T)$ is in MJ cm^{-1} nsec^{-1} keV^{-1}. In this formula and below, m_e is the electron mass, e is the electron charge, and k is the Boltzmann constant. Thus, for an actual high-temperature plasma one finds [2]

$$\kappa(T)=\kappa_L(T)\varepsilon\,\delta_T\approx\frac{1.301\,30\times10^{-2}T_e^{5/2}}{(1+0.290\,698Z)\ln\Lambda_{ei}}.\tag{7}$$

Equation (7) has approximately 10% accuracy [8]. The Coulomb logarithm Λ_{ei} can be written in the form (see, e.g., [2])

$$\ln\Lambda_{ei}=\ln\left[\frac{3}{2}\left(\frac{\bar{A}k^3T^3}{\bar{Z}\pi N_A\rho}\right)^{1/2}\frac{1}{Z+(1/2\alpha)(3kT/E_e)^{1/2}}\right],\tag{8}$$

where N_A is the Avogadro constant, α is the fine structure constant, and $E_e=m_ec^2$ is the electron rest energy. Also, for the considered mixtures $Z=3-y$. Now, by using the numerical values for the constants [9] in Eq. (8), one finds

$$\ln\Lambda_{ei}=\ln\left[\left(\frac{\bar{A}}{\bar{Z}}\right)^{1/2}\left(\frac{T^3}{\rho}\right)^{1/2}\frac{631.121\,59}{(Z+16.6018T)^{1/2}}\right].\tag{9}$$

Note also that only values of $\ln\Lambda_{ei}$ larger than 1 have physical meaning. So, if from the last equation it follows that (at some temperatures and densities) the Coulomb logarithm $\ln\Lambda_{ei}<1$, then it is assumed that $\ln\Lambda_{ei}=1$.

The competing (relatively slow) detonation expansion has the velocity $V_D(T)$, which in the general case is represented in the form $V_D=a\sqrt{T}$. An analytical expression for the factor a can be found, for example, by applying the so-called strong explosion approximation [10]. Below, we shall use the very strong explosion approximation, which provides the following, relatively simple expression for $V_D(T)$ (for more details, see, e.g., [6]):

$$\begin{aligned}V_D=a\sqrt{T}&=\sqrt{\frac{3\pi(\gamma+1)^2(\gamma-1)\mathcal{C}}{8(3\gamma-1)}}\sqrt{T}\\&=6.357\,08\times10^{-2}\sqrt{1-\frac{y}{5}}\sqrt{T},\end{aligned}\tag{10}$$

where $\gamma=\frac{5}{3}$, V_D is in cm nsec^{-1}, $\mathcal{C}$ is in MJ g^{-1} keV^{-1}, and T is in keV.

The energy release function $q(x,T)$ at $T\leqslant 15$ keV for the [D]:y[T]:$(1-y)$[^{3}He] mixture can be represented in terms of such two known functions: one for the equimolar (or 1:1) DT mixture $q_{dt}(x,T,\rho_0)$ and the other for pure deuterium $q_{dd}(x,T,\rho_0)$. The explicit expression for $q(x,T,\rho_0)$ takes the form

$$q(x,T,\rho_0)=\frac{2}{5}(1-y)^2 q_{dd}(x,T)+\left(\frac{2}{5}y+\frac{3}{5}y^2\right)q_{dt}(x,T) - \frac{C\sqrt{T}}{1+c\sqrt{\rho_0 x}T^{-7/4}}, \tag{11}$$

where the last (negative) term represents high-temperature bremsstrahlung loss. The denominator in this term corresponds to the correction for partial absorbtion of the emitted x-ray radiation in the hot, reaction zone. In fact, Eq. (11) is based on the assumption that the (d,t) reaction is significantly faster in all cases considered here than any of the competing (d,d) and (t,t) reactions. Moreover, since $y\leqslant 1$ in all present cases, the (t,t) reaction can be ignored, i.e., thermonuclear burn-up in excess deuterium. Briefly, this means that in the mixtures considered all tritium nuclei will react with an equal number of deuterium nuclei, and then the remaining deuterium nuclei will react with each other. Also, the tritium nuclei arising in one of the (d,d) reaction channels are assumed to instantaneously react with an equal number of deuterium nuclei. Such an approximation has good accuracy, since for the temperatures considered ($4\leqslant T \leqslant 15$ keV) the $\overline{\sigma v}_{dt}$ values (see below) are $\approx$59–75 times larger than the corresponding $\overline{\sigma v}_{dd}$ and $\overline{\sigma v}_{tt}$ values for both competing (d,d) and (t,t) reactions.

The $q_{dt}(x,T)$ and $q_{dd}(x,T)$ energy release functions in Eq. (11) can be written in the following general form:

$$q_{ij}(x,T)=s_{ij}(x,T)\left(1+B_{ij}\frac{b_{ij}x}{1+b_{ij}x}\right), \tag{12}$$

where $ij=dt$ or dd and B,b are the numerical constants. $s_{ij}(x,T)$ is the so-called short or ionic energy release function, which corresponds to the energy gain produced only by positive high-energy ions arising in the thermonuclear reactions. The small correction ($\simeq B$) represents the additional energy gain that neutrons (from thermonuclear reactions) leave inside the hot zone. The numerical values of the B,b, and c constants were chosen from [11] (see also [3]). The coefficient C in Eq. (11) can be written in the form (see [2,4])

$$C=116.414\,28\left(1-\frac{y}{3}\right)\left(1-\frac{3y}{5}\right). \tag{13}$$

The short energy release function $s_{ij}(x,T)$ is directly related to the corresponding $\overline{\sigma v}_{ij}$ value, e.g., for the 1:1 DT mixture and dense deuterium D_2 one finds (for more details see, e.g., [4,11])

$$s_{dt}(x,T)=8.176\,11\times 10^{18}\overline{\sigma v}_{dt},$$

$$s_{dd}(x,T)=3.039\,79\times 10^{19}\overline{\sigma v}_{dd}, \tag{14}$$

respectively. Here, both $s_{dt}(x,T)$ and $s_{dd}(x,T)$ values are given in MJ cm^3 g^{-2} nsec^{-1}. Also, in these formulas σ_{ij} is the ij fusion reaction cross section, v is the relative velocity of the reacting i and j nuclei, and $\overline{\sigma v}_{ij}$ designates the averaged (dimensionless) value over the whole range of relative velocities. Also, in the second equation for the (d,d) reaction all reactions with the ^{3}He nuclei formed are ignored. Note that, if thermonuclear burn-up proceeds in the presence of very intense neutron fluxes, the numerical factor before $\overline{\sigma v}_{dd}$ must be $4.631\,91\times 10^{19}$. However, this case is not of interest for our present study.

Both $\overline{\sigma v}_{dt}$ and $\overline{\sigma v}_{dd}$ values depend significantly upon temperature T. Here, for these values we use formulas from [11]. Now we can solve the burn-up equation, Eq. (2), numerically. The main goal is to find the minimal (or critical) value of the burn-up parameter $x_c=\rho_0 r_c$ at which thermonuclear burn-up still propagates in the considered [D]:y[T]:$(1-y)$[^{3}He] mixture. In fact, Eq. (2) has been rewritten to the following integral equation form [11]:

$$T(x)=T_0\left(\frac{x_0}{x}\right)^3+\frac{1}{\mathcal{C}x_0^3}\int_{x_0}^{x}dz\,\frac{q(z,T(z))z^3}{V_{\max}(z,T(z))}. \tag{15}$$

The critical burn-up parameter $x_c=\rho_0 r_c$ has been determined from this equation as a function of y,T, and ρ_0. The minimal energy $\mathcal{E}_c$ required to produce thermonuclear ignition can easily be evaluated in terms of the known x_c parameter: $\mathcal{E}_c=(4\pi/3)x_c^3\rho_0^{-2}\mathcal{C}T$, where $\mathcal{E}_c$ is in MJ and ρ_0 and T are the initial density and temperature, respectively.

In the present study the following values of the tritium concentrations y have been considered: 1, 0.75, 0.5, 0.2, 0.1, 0.05, 0.02, 0.01, and 0.0 (see results in Table I). In all cases in Table I the initial density ρ_0 equals 100 g cm^{-3}. The temperatures T in the present study are bounded between 4 keV and 15 keV. Note that for all considered densities such temperatures are significantly higher than the so-called equivalent Fermi temperature T_{ef} [12]. For the [D]:y[T]:$(1-y)$[^{3}He] mixture the equivalent Fermi temperature T_{ef} takes the following form:

$$T_{ef}\ (\text{keV})=\frac{2}{5}\left(\frac{3N_A}{8\pi}\right)^{2/3}\frac{h^2}{2m_e}\left(\frac{\rho_0}{5}\right)^{2/3}(1.5-0.5y)^{2/3} =4.168\,60\times 10^{-3}\rho_0^{2/3}\left(1-\frac{y}{3}\right)^{2/3}, \tag{16}$$

where N_A, h, and m_e are the Avogadro number, Planck constant and electron mass, respectively [9]. Here, the maximal initial density $\rho_0\leqslant 1\times 10^3$ g cm^{-3} (see below). Therefore, $T_{ef}\leqslant 0.416\,86$ keV$\ll$4 keV, i.e., the appropriate correction related to the electron degeneracy is negligible. This indicates that low-temperature electron degeneracy cannot affect our present solution. In the opposite case, i.e., when $T_{ef}\approx 4$ keV, the corresponding disassembly (or rarefaction) wave will pass the thermonuclear burning wave, and our present calculation cannot be used.

As follows from Table I, the critical value of the burn-up parameter x_c grows very rapidly as the molar concentration

TABLE I. The critical values of the burn-up parameter x_c (in g cm^{-2}) for various molar tritium concentrations y in $[\mathrm{D}]{:}y[\mathrm{T}]{:}(1-y)[{}^3\mathrm{He}]$ mixtures and temperatures T (in keV). Here, $\rho_0 = 100$ g cm^{-3} everywhere.

	y								
T	1	0.75	0.5	0.25	0.1	0.05	0.02	0.01	0.00
4	2.831	5.477	12.82	50.08	294.1	910.7	2578	4135	7447
5	1.757	3.398	7.957	29.55	161.1	513.0	1571	2659	5230
6	1.216	2.333	5.475	20.05	104.6	341.4	1125	2002	4291
7	0.9105	1.725	4.028	14.70	74.69	249.2	879.1	1638	3795
8	0.7223	1.351	3.120	11.34	56.61	192.6	724.1	1407	3494
9	0.5985	1.106	2.522	9.098	44.73	154.6	617.7	1247	3291
10	0.5126	0.9386	2.112	7.532	36.49	127.6	540.5	1131	3144
11	0.4505	0.8187	1.821	6.409	30.56	107.7	482.1	1043	3034
12	0.4041	0.7301	1.608	5.584	26.20	92.52	436.8	974.9	2950
13	0.3684	0.6626	1.448	4.965	22.92	80.82	401.0	922.3	2884
14	0.3403	0.6099	1.324	4.491	20.42	71.67	372.3	881.2	2833
15	0.3177	0.5680	1.226	4.121	18.47	64.44	349.1	849.1	2794

of the remaining tritium y decreases. In general, it is assumed (see, e.g., [3] or [11]) that thermonuclear burn-up can be produced in practice only if $r_c \leqslant 1$ cm, or, in other words, $x_c \leqslant \rho_0$. In terms of this approximate criterion, one can easily determine that the density $\rho_0 = 100$ g cm^{-3} is too low to produce actual thermonuclear ignition in $[\mathrm{D}]{:}y[\mathrm{T}]{:}(1-y)\times[{}^3\mathrm{He}]$ mixtures at $T \leqslant 6$ keV, if $y \leqslant 0.05$. Thermonuclear ignition in such conditions requires significantly higher compressions (or larger densities). This can be seen from Table II, where the dependence $x_c(\rho_0)$ is shown for $T = 5, 10$, and 15 keV and for $y = 0.5, 0.1, 0.02$, and 0.00. The critical values of the burn-up parameter x_c for the $[\mathrm{D}]{:}y[\mathrm{T}]{:}(1-y)[{}^3\mathrm{He}]$ mixture at $y = 0.02$ and 0.00 are significantly larger than for pure deuterium [11] in the same conditions. This is related to the very high bremsstrahlung loss in $[\mathrm{D}]{:}y[\mathrm{T}]{:}(1-y)[{}^3\mathrm{He}]$ mixtures with $y \approx 0$.

Table III shows the dependence of the critical burn-up parameter x_c upon time t (for temperatures $T = 5, 10$, and 15 keV). In fact, in this table the dependence of $\log_{10} x_c$ upon t is presented. The concentration of remaining tritium nuclei y decreases exponentially with time t, i.e., $y(t) = y(0)\exp(-\alpha t)$, where $y(0) = 1$ and α is the inverse half-life of the tritium, i.e., $\alpha^{-1} = 12.5$ yr. In Table III time t is in years (with $\Delta t = 10$ yr) and also $\rho_0 = 100$ g cm^{-3} (everywhere).

It should be mentioned that at the relatively low temperatures considered ($T \leqslant 15$ keV) in the $[\mathrm{D}]{:}y[\mathrm{T}]{:}(1-y)[{}^3\mathrm{He}]$ mixtures the thermonuclear burning wave always propagates as a high-temperature detonation wave, i.e., thermonuclear burn propagates by gas-dynamic detonation. The competing electron thermal conduction wave is relatively slow for such temperatures ($T \leqslant 15$ keV). Furthermore, for $T \leqslant 15$ keV only the two (d,t) and (d,d) thermonuclear reactions can be included. This simplifies the present analysis significantly. In contrast to this, for higher temperatures (e.g., $T \geqslant 20$ keV) both the $(d,{}^3\mathrm{He})$ and $(n,{}^3\mathrm{He})$ reactions must be considered. Likewise, for $T \geqslant 30$ keV the isothermal approximation (i.e., $T_i = T_e = T$) used above becomes quite inaccu-

TABLE II. The critical values of the burn-up parameter x_c (in g cm^{-2}) for various densities ρ_0 (in g cm^{-3}), molar tritium concentrations y in the $[\mathrm{D}]{:}y[\mathrm{T}]{:}(1-y)[{}^3\mathrm{He}]$ mixtures, and temperatures T (in keV).

		ρ_0						
y	T	1	5	10	50	100	500	1000
0.5	5	25.79	17.27	14.39	9.437	7.957	5.670	5.061
0.5	10	2.442	2.381	2.340	2.196	2.112	1.880	1.777
0.5	15	1.294	1.283	1.276	1.246	1.226	1.159	1.123
0.1	5	11520	2348	1201	279.4	161.1	58.41	42.36
0.1	10	537.1	167.1	110.4	49.20	36.49	19.86	15.86
0.1	15	35.23	30.39	27.81	21.27	18.47	12.73	10.74
0.02	5	148800	29830	14960	3059	1571	376.6	223.8
0.02	10	48600	9765	4912	1027	540.5	144.5	91.12
0.02	15	29020	5857	2961	642.0	349.1	104.5	68.89
0.00	5	510600	102220	51170	10340	5230	1144	629.8
0.00	10	307290	61520	30790	6217	3144	684.3	374.9
0.00	15	273100	54670	27370	5525	2794	607.4	332.2

TABLE III. The dependence $\log_{10} x_c$ upon time t (in yr) for $[\mathrm{D}]:y[\mathrm{T}]:(1-y)[^3\mathrm{He}]$ mixtures ($\rho_0 = 100$ g cm^{-3} everywhere) and temperatures T (in keV).

	T		
t (yr)	5	10	15
0	0.2448	−0.2902	−0.4979
10	0.9921	0.4140	0.1733
20	1.6424	1.0369	0.7681
30	2.2831	1.6373	1.3381
40	2.8377	2.2621	1.9820
50	3.2314	2.7810	2.6016
60	3.4709	3.1203	3.0084
70	3.5997	3.3123	3.2329
80	3.6634	3.4104	3.3463
90	3.6934	3.4575	3.4004
100	3.7072	3.4794	3.4254

rate. This means that our present method can be used to determine the critical burn-up conditions only at relatively low temperatures. The final stage of the burn-up process, when temperatures and thermal conductivity are very high, requires a separate investigation.

In conclusion, it should be noted that our present method can be used to predict low-temperature thermonuclear burn-up in various thermonuclear fuels, that contain deuterium and tritium. In fact, it is crucial for this method that thermonuclear burn propagates from the central hot spot to the rest of the fuel, which is, in general, slightly heated, but strongly compressed. Such targets are of great interest in applications, since they have very high amplification coefficients. The generalization of the present method to the cases of cylindrical and plane symmetries is straightforward. The critical conditions for this problem (spherical symmetry) have been extensively studied in the literature, primarily for the equimolar DT mixture (see, e.g., [2,3,8,11,13–15]). The results of these studies are in good agreement with each other. In general, the agreement becomes even better for higher temperatures. Some deviations for low temperatures are related to the leading mechanism of thermonuclear burn wave propagation. In [8] it was assumed that the thermonuclear burn wave propagates by electron thermal conductivity, while in [13–15] such propagation was related mainly to thermonuclear α particles. The supersonic, gas-dynamic propagation for the thermonuclear burn wave was discussed in [3] and later in [11].

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[1] Tritium β^- decay can be written symbolically in the form $\mathrm{T} = {}^3\mathrm{He} + e^- + \overline{\nu} + 18.57$ keV, where ^{3}He is the daughter nucleus, e^- is the emitted electron, and $\overline{\nu}$ is the electron antineutrino. In fact, such a process proceeds in molecules, rather than in atoms. However, we will not discuss this process in detail at present.

[2] G.S. Fraley, E.L. Linnebur, R.K. Mason, and R.L. Morse, Phys. Fluids **17**, 474 (1974).

[3] E.V. Avrorin, L.P. Feoktistov, and L.I. Shibarshov Fiz. Plasmy, **6**, 965 (1980) [Sov. J. Plasma Phys. **6**, 527 (1980)].

[4] S. Glasstone and R.H. Lovberg, *Controlled Thermonuclear Reactions* (Van Nostrand, Princeton, N.J., 1960).

[5] L. Spitzer, *Physics of Fully Ionized Gases* (Interscience, New York, 1962).

[6] A.M. Frolov, Can. J. Phys. **26**, 33 (1994).

[7] Here we assume that the difference between the proton and neutron masses is negligible and also that the mass defect for all considered nuclei equals zero exactly. Both these approximations are traditional in thermonuclear problems, and have been widely used in earlier work (see, e.g., [2] or [4]).

[8] K.A. Brueckner and S. Jorna, Rev. Mod. Phys. **46**, 325 (1974).

[9] E.R. Cohen and B.N. Taylor, Phys. Today **51** (8), 9 (1993).

[10] V.P. Korobeinikov, *Problems of Point-Blast Theory* (American Institute of Physics, New York, 1991).

[11] A.M. Frolov, Plasma Phys. Controlled Fusion **40**, 1417 (1998); **41**, 759 (1999), and references therein.

[12] L.D. Landau and E.M. Lifshitz, *Statistical Physics* (Pergamon London, 1958), Chaps. V and XI.

[13] J.S. Clarke, H.N. Fisher, and R.J. Mason, Phys. Rev. Lett. **30**, 89 (1973).

[14] E.G. Gamalii, S.Yu. Guskov, O.N. Krokhin, and V.B. Rozanov, Fiz. Plasmy **1**, 904 (1975) [Sov. J. Plasma Phys. **1**, 494 (1975)].

[15] J. Lindl, Phys. Plasmas **2**, 3933 (1995), and references therein.